

Impact Evaluation in Firms and Organizations

With Applications in R and Python

Chapter 7: Regression Discontinuity Designs

7.1 Sharp and Fuzzy Regression Discontinuity Designs

7.2 Behavioral Assumptions and Methods

Basic idea of the Regression Discontinuity Design

- Intervention is determined by passing a threshold in a covariate, also called the running variable
- All units with a running variable value above a certain threshold receive the intervention and the others do not

Example

Evaluating customer retention

- Internet provider offers a discount to prevent customer churn
- Discount assignment is determined by churn probability, which serves as running variable
- Discount offered only to customers with churn probability at least 50%

Example

Evaluating customer retention (continued)

- Customers at or above 50% receive the discount, while the others do not
- Intervention switches from 0 to 1 at the threshold, i.e., the threshold determines intervention status
- Discontinuity in the intervention status at the threshold of the running variable
- Discontinuity, or jump, in the intervention at the threshold is the basis of the regression discontinuity design

Local impact evaluation

- Impact is assessed by comparing outcomes of treated and untreated units around the threshold
- Assumes units just above and below the threshold share characteristics
- Allows estimation of the intervention effect for units close to the cut-off

Example

Evaluating customer retention (continued)

- Compare customers at 50% churn probability with those at 49%
- Groups around the threshold are similar and thus comparable
- Mimics an experimental setting with comparable treatment and control groups

Need to avoid manipulation

- Subjects above and below the threshold must be comparable
- Subjects influencing the running variable to cross the threshold and thus receive the intervention create systematic differences
- Such manipulation leads to invalid comparisons

Example

Strategic behavior in practice

- Loyalty card offered to customers exceeding an expenditure threshold
- Customers aware of this may adjust spending to surpass it
- Groups just above and below the threshold may differ in purchasing inclination

Regression Discontinuity Design

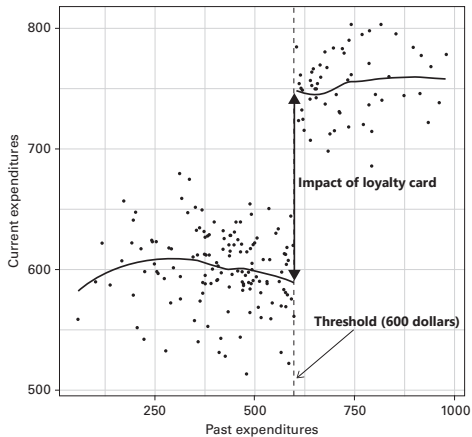


Figure 7.1: Regression discontinuity design

Time-based threshold assignment

- Intervention occurs at a specific point in time, e.g., third week of September
- Assignment of intervention determined by whether observations fall before or after this date
- Compare outcomes in the third week (right after intervention) to the second week (right before)

Ensuring validity

- Before–after comparison must not be affected by factors other than the intervention, e.g., weather changes may affect sales
- Intervention must not be anticipated, e.g., customers may already know the upcoming pricing change

Partial compliance around the threshold

- Sharp regression discontinuity design requires everyone above the threshold to be treated, and everyone below to be untreated
- Individuals may only partially comply, e.g., some subjects above the threshold do not receive the intervention

Example

Partial compliance in practice

- Students meeting a test score threshold to enter an education program may not join it
- Customers meeting a purchasing threshold for a loyalty card may not acquire it
- Both situations lead to treatment probabilities changing at the threshold rather than jumping from zero to one

Existence of compliers

- Fuzzy regression discontinuity design approach evaluates impact for compliers who switch treatment at the threshold
- Compliers change intervention status when moving from below to above the cut-off
- Approach thus requires existence of such compliers at the threshold

Connection to instrumental variables

- Passing the threshold serves as an instrument for the intervention
- Passing cut-off must not reduce intervention take-up for any subject (weak monotonicity condition)

7.1 Sharp and Fuzzy Regression Discontinuity Designs

7.2 Behavioral Assumptions and Methods

Formalizing the Sharp Regression Discontinuity Design

Running variable and threshold

- R denotes the running variable and r_0 the threshold
- Intervention defined as $D = 1$ if $R \geq r_0$
- Subjects with $R < r_0$ have $D = 0$

Sharp discontinuity in treatment

- All subjects switch from $D = 0$ to $D = 1$ at the threshold r_0
- Treatment assignment determined solely by passing the cut-off

Continuity assumption

- No abrupt shifts in factors other than D at the threshold
- Background characteristics affecting Y are similar around the threshold
- Known as continuity of covariates in statistical terms

Local Average Treatment Effect at the Threshold

Defining the local ATE

- Local effect focuses on subjects at the threshold r_0
- Aim to estimate $\Delta_{r_0} = E[Y(1) | R = r_0] - E[Y(0) | R = r_0]$
- Formally, the local ATE at the threshold r_0 is defined as

$$\Delta_{r_0} = \lim_{\epsilon \rightarrow 0} E[Y | R \in [r_0, r_0 + \epsilon)] - \lim_{\epsilon \rightarrow 0} E[Y | R \in [r_0 - \epsilon, r_0)] \quad (7.1)$$

- Compare outcomes within a small window $\epsilon > 0$ around r_0 which ensures units are nearly identical in running variable values
- Measures impact for units whose running variable equals the cut-off

Plot on the Discontinuity

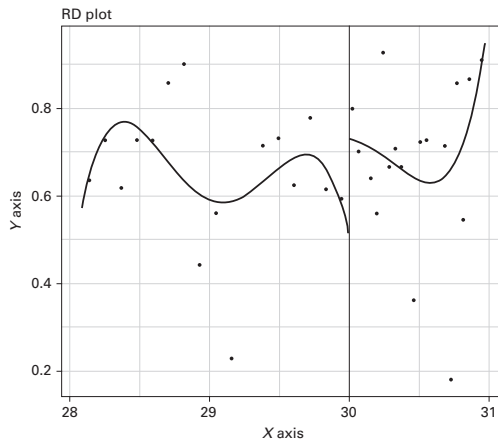


Figure 7.2: Plot on the discontinuity

Potential outcomes across the running variable

- $E[Y(1) | R]$ represents average potential outcome with intervention at each value of R
- $E[Y(0) | R]$ represents average potential outcome without intervention at each value of R

Given intervention switches from $D = 0$ to $D = 1$ at the threshold r_0

- Observed potential outcomes
 - $E[Y(1) | R]$ observed only for $R \geq r_0$, where $D = 1$
 - $E[Y(0) | R]$ observed only for $R < r_0$, where $D = 0$
- Unobserved potential outcomes
 - $E[Y(1) | R]$ is unobserved for $R < r_0$
 - $E[Y(0) | R]$ is unobserved for $R \geq r_0$

Sharp Regression Discontinuity Design

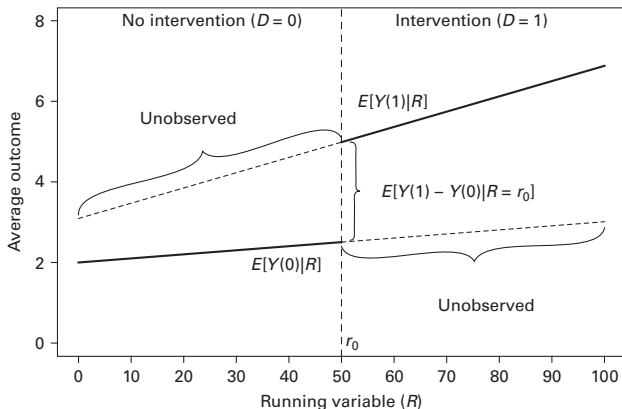


Figure 7.3: Sharp regression discontinuity design

Compliance of subjects

- Not all subjects above the threshold may receive the intervention
- Some subjects below the threshold may still receive it
- In such cases, treatment assignment is not perfectly determined by the cut-off

Impact evaluation for compliers

- Fuzzy regression discontinuity design evaluates impact only for subjects switching treatment at the threshold, i.e., the compliers
- Assumes that compliers exist who switch from no intervention to intervention at the threshold r_0
- Assumes monotonicity such that no subjects exist who switch from intervention to no intervention at the threshold r_0

Threshold-based instrument

- Define $Z = 1$ if $R \geq r_0$ and $Z = 0$ if $R < r_0$
- D denotes receiving the intervention, which may differ from the threshold-based instrument Z

First-stage effect at the threshold γ_{r_0}

- Measures impact of Z on D at the threshold r_0

$$\gamma_{r_0} = \lim_{\epsilon \rightarrow 0} E[D \mid R \in [r_0, r_0 + \epsilon)] - \lim_{\epsilon \rightarrow 0} E[D \mid R \in [r_0 - \epsilon, r_0)] \quad (7.2)$$

Intention-to-treat effect θ_{r_0}

- Measures impact of Z on Y at the threshold r_0

$$\theta_{r_0} = \lim_{\epsilon \rightarrow 0} E[Y \mid R \in [r_0, r_0 + \epsilon)] - \lim_{\epsilon \rightarrow 0} E[Y \mid R \in [r_0 - \epsilon, r_0)]$$

Local ATE at the threshold

- Local ATE is obtained by scaling the intention-to-treat effect by the first-stage effect
- γ_{r_0} measures impact of passing the threshold on intervention D
- θ_{r_0} measures impact of passing the threshold on outcome Y
- Formally, the local ATE is defined as

$$\Delta_{D(1)=1, D(0)=0, R=r_0} = \frac{\theta_{r_0}}{\gamma_{r_0}} \quad (7.3)$$

Localized nature of the effect

- Effect applies only to compliers ($D(1) = 1, D(0) = 0$) at the threshold r_0
- Focuses on units switching their treatment status when passing the threshold r_0

The Fuzzy Regression Discontinuity Design

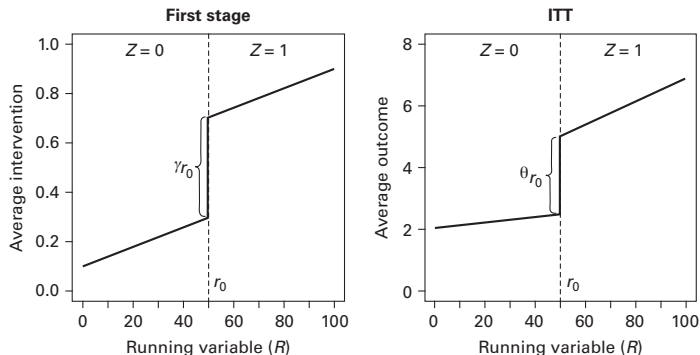


Figure 7.4: The fuzzy regression discontinuity design

Bias-variance trade-off

- Data window in terms of distance from the threshold, also called bandwidth, is an important parameter
- Narrower window improves comparability and reduces estimation bias but increases variance due to fewer observations

Selecting an optimal bandwidth

- Choose bandwidth that optimally trades off bias and variance
- Aim: minimize overall error
- Data-driven methods compute optimal bandwidth for a given sample

Assessing manipulation around the threshold

- Behavioral assumption requires no manipulation of the running variable
- Examine whether similar numbers of subjects appear just below and above the threshold using a density check
- Large imbalance suggests manipulation or bunching

Example

Loyalty card leading to bunching

- Loyalty card given above a \$600 expenditure threshold
- Customers aware of the rule may shift spending to exceed \$600
- Excess mass above the threshold indicates manipulation, and thus non-comparable groups

Similarity of observed characteristics around the threshold

- Check whether covariates X are similar around the threshold
- Lack of similarity suggests that groups around the threshold may not be comparable

Example

Loyalty card leading to systematic differences

- Customers just above \$600 may differ in characteristics from those just below
- Violates the assumption of comparable groups near the cut-off